A TEST OF A PERSPECTIVE THEORY OF GEOMETRICAL ILLUSIONS

DOMINIC W. MASSARO and NORMAN H. ANDERSON, University of California, San Diego

Abstract. Three-dimensional Müller-Lyer figures were used in three experiments to test a perspective theory of geometrical illusions. The real depth cues were in one of two orientations, designed either to support or to oppose the action of the perspective depth cues hypothesized by the perspective theory. If any illusion is obtained with these three-dimensional figures, the theory implies, its magnitude should be different in the two orientations. In fact, a substantial illusion was obtained, but the difference between the two orientations was small and opposite to the prediction of perspective theory.

Perspective explanations of geometrical illusions have a long history, but only recently has this approach received the conceptual clarification needed to open it to experimental analysis. Gregory's formulation of perspective theory rests on two main assumptions. First, geometrical-illusion figures such as the Müller-Lyer ones in Figure 1ab are considered two-dimensional projections of three-dimensional figures; thus, they contain perspective cues for depth. Second, these cues are assumed to trigger a constancy-scaling mechanism that corrects for decrease in the retinal image with increasing distance. Those parts of the figure that would be more distant in the three-dimensional representation are enlarged and those that would be closer are diminished, in accord with size constancy.

This version of perspective theory has been applied especially to

Received for publication April 27, 1970. The first author, who is now at the University of Wisconsin, was supported by a postdoctoral fellowship, MH-39369-02, from the National Institute of Mental Health. Support was also provided by a National Science Foundation grant, GB-6666, to the second author. The authors thank Susan Keeve for her help.

the planar Müller-Lyer figures in Figure 1ab. As Gregory notes, these can be seen as two-dimensional projections of different objects depending on the direction of the wings. Figure 1a could be the projection of the near vertical corner of a building or of a box. The wings represent the lines of the top and bottom of the object receding into the distance. Size constancy, corresponding to the adjustment in apparent size that keeps the object parallelepipedal, would decrease the length of the main axis of the figure. In Figure 1b, which could represent the projection of the far corner of a room, the constancy scaling would analogously expand the apparent size of the main axis.

Gregory's formulation has provoked numerous criticisms, most of which attempt to show that perspective theory does not account for certain illusions, such as the dumbbell version of the Müller-Lyer figure. However, perspective cues could still be operative in some illusions, even though additional factors might also be involved. Perspective theory may thus have a valid role in the explanation of illusions, and the present experiments attempted to get

---

Fig. 1. (a) and (b) Typical two-dimensional Müller-Lyer figures; (c) three-dimensional figure with the wings oriented away from the observer, (d) wings oriented toward the observer; (e) stick figure with the wings oriented away from the observer, and (f) wings oriented toward the observer.

---

3 Gregory, Seeing in depth (n. 2).
a direct test of it by varying the depth cues of three-dimensional Müller-Lyer figures as is illustrated in Figure 1. The question of interest is how the real depth of the wings affects the normal illusion. For example, one critical comparison is between Figure 1c and 1d. These are geometrically identical, except that the wings extend away from the observer in Figure 1c, toward the observer in Figure 1d.

Oddly enough, the most direct interpretation of perspective theory predicts no illusion for three-dimensional Müller-Lyer figures. Since they are easily seen in correct depth perspective, the hypothesized constancy-scaling mechanism should not be triggered inappropriately, and hence there should be no illusion. But this is wrong, as can perhaps be seen by comparing Figure 1e and 1f, as demonstrated below, and as reported earlier by Lucas and Fisher for a three-dimensional Poggendorff illusion. It is necessary, therefore, to search for the perspective cues that must exist if perspective theory is to account for the illusion in the three-dimensional figures.

One possible application of perspective theory would attribute the illusion to inappropriate constancy scaling triggered by the three-dimensional form. Thus, Figure 1c represents the corner of a building no less than Figure 1a. Similarly, Figure 1d represents the corner of a room no less than Figure 1b. Direct transposition of the two-dimensional theory would then predict that the central axis should appear shorter in Figure 1c than in Figure 1d. Indeed, this interpretation would require that Figure 1c appear shorter than even the control version of Figure 1d with 90-deg. wings. As Figure 1 suggests, and as will be seen below, this is not true.

There seems to be only one remaining possibility, namely, that the illusion is produced by certain depth cues of the two-dimensional projections of the figures. The retinal image itself is two-dimensional, so the correct perception, or real depth, must be constructed from the available depth cues. These include the usual cues to real depth, such as convergence, stereopsis, interposition, and so forth. And, if perspective theory is correct, they also include the perspective features of the two-dimensional retinal image that are assumed to produce constancy scaling.

There are, then, two sets of depth cues, one ‘real’ and one hypothesized by perspective theory. These two sets of cues can act in concert or in opposition. If the real depth cues oppose the perspective

---

cues, then the illusion should be less than if they act in concert. Accordingly, the figures shown in Figure 1cd provide a critical test. In Figure 1c, the wings slope away from the observer, and, therefore, the depth cues of the two dimensional projections disagree with the actual depth; in Figure 1d, they agree. Since the perception of depth is dependent upon all available cues, perspective theory predicts a larger illusion for Figure 1d than 1c. But if perspective theory is wrong, both figures should give essentially the same effect. It is on this logic that the experiments were designed.

**METHOD**

*Stimuli for Experiments I and II.* Examples of the stimuli are shown in Figure 1cd. They were constructed from stiff off-white cardboard, first cut to an areal version of the Müller-Lyer figure, and then folded along the central axis so that the two sides met at right angles. The width of the side was 32 mm. in all figures.

Twenty figures were used, in a $2 \times 2 \times 5$ design. There were two orientations, away from or toward the $S$, as in Figures 1c and 1d respectively. Two lengths of central axis were used, 127 and 152 mm. And the edge of the side, or wing, met the central axis at one of five angles: 45, 60, 90, 120, and 135 deg. The angles shown in Figure 1cd are both at 135 deg.

These figures were mounted in wooden holders 25 cm. long, with the $5 \times 3$ cm. cross section shaped at one end to a 90-deg. angle to accommodate the figures. These holders rested on a table at eye level about 100 cm. from the $S$, with the figures projecting 10 cm. beyond the edge of the table. Two lamps were used in addition to the overhead fluorescent to provide uniform illumination on the figures. The $E$ sat at the side of the table, presenting the stimuli and recording the responses. The $S$s were instructed to judge the apparent length of the central axis, and two practice trials were given.

*Procedures for Experiments I and II.* In Experiment I, the figures were presented in a horizontal position. The $S$ made his response by adjusting the length of a motorized white tape displayed horizontally on a black panel about 30 cm. below and 10 cm. to the side of the stimulus figure. Two groups of 8 $S$s from introductory courses were used. Each $S$ judged the 10 figures with one length of central axis. These 10 figures were judged 10 times in successive blocks, each block given in a different random order for each $S$.

In Experiment II, the figures were presented in a vertical position, as in Figure 1. The 16 $S$s judged all 20 figures, 5 times each in separately randomized order in five successive blocks. The response was made by marking off on a 205-mm. black vertical line beginning 11 mm. below the top edge of a sheet of white paper.

*Stimuli for Experiment III.* This experiment used black stick figures, 150 mm. long and 3 mm. thick, with 50-mm. wings. The wings were displaced 3 mm. in from the ends of the central axis to avoid any possible doubt about the location of the ends of the central axis. The figures were presented verti-
cally on a knitting needle 7 cm. from a white background wall, about 110 cm. from the S.

The 5 three-dimensional stimuli were constructed so that they could fit in the corner of a room, the central axis in the vertical and the wings flat on the two walls. The wings had angles of 30, 60, 90, 120, and 150 deg. with the central axis. Figure 1ef shows the 60-deg. angle oriented away from, and the 120-deg. angle oriented toward, the camera. Two two-dimensional stick figures, analogous to those in Figure 1ab, and a control without wings, were also used.

Procedure for Experiment III. The general procedure was similar to that of Experiment II, and Ss made their response by marking a line on a response sheet in the same way. The 5 three-dimensional figures were presented in both orientations, so there were 13 possible stimuli altogether. These were presented 5 times each in successive blocks. The Ss were 20 students from an introductory course.

Statistical analysis. Complete three-way or four-way analysis of variance was applied to the data of each experiment, including trial blocks as a factor. The main results are obvious in the graphs, and only those that require special mention are specifically discussed.

RESULTS

Experiments I and II. The mean judgments in Experiments I and II are shown in the left and right panels of Figure 2. The upward trend of the curves shows that the apparent length of the constant central axis was markedly influenced by wing angle. Thus, the Müller-Lyer illusion was obtained with these three-dimensional figures.

Since the illusion did occur, the critical test of perspective theory is given by the comparison between the open and closed circles in each panel. If perspective theory is correct, the open circles, which correspond to the corner of a building or a box, should lie below the closed circles. Exactly the opposite was seen in both experiments. The mean difference between the inward and outward orientations was .64 mm. in Experiment I, and 1.64 mm. in Experiment II. The former difference was not significant \[ F (1, 14) = 3.22 \], but the latter was \[ F (1, 15) = 21.71 \]. Both experiments, therefore, contradict perspective theory.

Experiment III. Although the first two experiments argue against perspective theory, no cause was seen for the opposite effect obtained in Experiment II. The slight difference in retinal angle of the total figure would actually favor perspective theory. This was small, but it was feared that it might reflect some peculiarity of the cardboard figures. If such an effect existed, it might mask a
real perspective effect. Accordingly, stick figures were used to eliminate any possible perceptual ambiguity about the endpoints of the central axis.

Figure 3 shows the mean judgments of the three-dimensional stick figures. The upward slope of the curve demonstrates that these stick figures also produced the illusion. In the critical comparison, however, the mean difference between the inward and outward orientation was only .17 mm., which did not approach significance. Thus, these data also contradict perspective theory.

**Miscellaneous results.** A few minor results from the analyses should also be reported. In Experiment I, there was a significant increase over trials in mean response, from 132.5 mm. in the first trial block to 139.6 mm. in the last trial block. This increase may have been caused by the response mechanism, which was somewhat inconvenient to control. At any rate, no such increase was obtained in the other two experiments, in which the line-marking response was employed.

In Experiment III, the two-dimensional stick figures also showed the illusion, the mean judgments being 128.7 and 135.7 mm. for
the inward and outward wings. In comparison, the mean judgment for the wingless control stick was 130.8 mm. These data show an asymmetry similar to the nonlinearity in Figure 3. Possibly, this reflects a contrast effect, as mentioned by Müller-Lyer.3

Experiment I showed a significant decrement in the magnitude of the illusion over trials. This result agrees with the practice decrement reported for two-dimensional figures.7 The illusion still persisted at the last trial block, however, and no decrement was found in Experiment II. Experiment III showed a small, but nonsignificant, decrement in the illusion for both the two- and three-dimensional stick figures.

DISCUSSION

The present three-dimensional Müller-Lyer figures appear to provide a critical test of Gregory's perspective theory of geometrical illusions (n. 2). If this theory is to account for the illusion obtained with three-dimensional figures, it presumably must invoke the same set of depth cues assumed to operate in the two-dimensional figures.

Fig. 3. Mean judged length as a function of the wing angle and the real depth perspective of the figures used in Experiment III

These cues are hypothesized to produce a constancy correction for implied distance that expands or contracts the apparent length of the central axis.

But three-dimensional figures also contain a second set of depth cues, cues that produce the perception of their real depth. The critical comparison is, then, between the same figure in its outward and inward orientations, as illustrated in Figure 1c and 1d respectively. In the outward orientation, the two sets of depth cues disagree; in the inward orientation, they agree. Perspective theory then implies that the size of the illusion should be less in the former case than in the latter. It should be noted that this is a direct test of the perspective/constancy-scaling hypothesis. Other factors might play a role, but if perspective has any effect at all, then it should have appeared in the relevant comparison. Contrary to perspective theory, no such effect was obtained.

Perspective theory can hardly argue that the hypothesized constancy scaling does not apply to the three-dimensional figure, for then the theory cannot account for the strong illusion that these figures give. A new explanatory principle would then be required, one that presumably would explain the two-dimensional illusion at the same time. The only theoretical alternative, then, would be to argue that the perception of length is completely independent of the perception of real depth. This would be sufficiently remarkable that it may merit some consideration. However, it is not consistent with the idea that the object has a unitary perception.

Strictly speaking, the present test applies only to Gregory’s perspective/constancy-scaling interpretation of the Müller-Lyer illusion. However, it would seem that the same logic would apply to the general perspective hypothesis of this illusion, regardless of the mechanism of its operation. How far these results may generalize to other illusions is uncertain. Our observations do indicate a constancy-scaling effect in the Necker cube, and Jeffrey has reported analogous effects in other reversible figures. On the other hand, Fisher has reported no effect when subjects assume different perspective orientations of the ordinary Müller-Lyer figure. For the standard illusion, therefore, it may be most profitable to seek the explanation elsewhere.

REFERENCES

There is another approach to the illusions that has nearly as long a history as perspective theory. These are the ‘total impression’ theories that emphasize judgmental processes rather than sensory-perceptual factors. Gregory has criticized the total-impression theories as being nontheories, incapable of definite prediction. More recently, however, two mathematical formulations of this type have been suggested. Restle and Merryman have applied adaptation-level theory to the analysis of certain illusions. Anderson has given a development based on integration theory and functional measurement. Both these formulations make quantitative predictions that have enjoyed some provisional success.

10 Woodworth (n. 1), 645.
11 Gregory (n. 1), 142–143.