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JUDGMENTAL MODEL OF THE EBBINGHAUS ILLUSION [1]

DOMINIC W. MASSARO [2] AND NORMAN H. ANDERSON

University of Wisconsin — *University of California, San Diego*

Two experiments studied the Ebbinghaus illusion as a function of four stimulus variables: the size of the context circles, the number of context circles, the distance between the context circles and the center circle, and the size of the center circle. The results provided a quantitative test of a judgmental model that considers the Ebbinghaus illusion to be comparative in nature. The context circles, then, serve as standards or yardsticks, and the center circle is judged partly relative to them. The model provided a reasonably good description of the magnitude of the illusion as a function of the several stimulus variables.

The center circle in Fig. 1 appears larger when surrounded by small circles, smaller when surrounded by large circles. This figure, known as the Ebbinghaus illusion, or Titchener circles, has seen relatively little systematic study. The present article reports two parametric studies done within a judgmental theory of stimulus integration (Anderson, 1970a, 1970b; Massaro & Anderson, 1970). The Ebbinghaus illusion is considered to be comparative: the surrounding context circles serve as standards or yardsticks, and the center circle is judged, in part, relative to them.

Comparative judgment of the Ebbinghaus figure is assumed to be a weighted average of "absolute" and relative factors. Let s be the absolute size of the center circle, corresponding to its perceived size with no context circles. Let s^* be the relative size of the center circle, judged with respect to a single context circle. Thus s^* will be larger (or smaller) than s if the center circle is larger (or smaller) than the context circle. Thus s^* represents a contrast effect.

The equation for judgment of the center circle in the presence of a single context circle is

$$J = w_1 s + w_2 s^*, \qquad [1]$$

where w_1 and w_2 are the weights of the two respective determinants of judged size. In an averaging model, the weights sum to unity so that Equation 1 may be rewritten as

$$J = ws + (1 - w)s^*. \qquad [2]$$

Since $1 - w$ is the weight given the relative size with a single context circle, the weight of k circles of similar size and placement is $k(1 - w)$. The judgment equation then becomes

$$J = [ws + k(1-w)s^*]/[w + k(1-w)], \qquad [3]$$

where the denominator is a normalizing factor that forces the relative weights of s and s^* to sum to unity for each value of k.

Equation 3 is just the set size function of integration theory for the case of simultaneous stimulus presentation (Anderson, 1968). It is a growth function of k, with asymptote s^*. (The complete form of Equation 3 would contain a fixed term in s but this may be neglected here since k cannot be very large in practice).

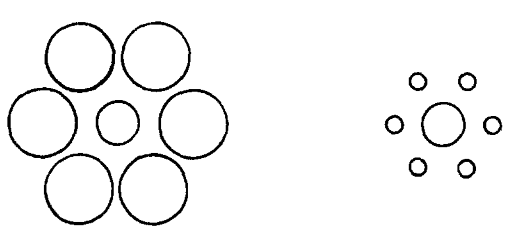

FIG. 1. Illustration of Ebbinghaus illusion. (Center circle has the same size in both panels.)

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[2] Requests for reprints should be sent to Dominic W. Massaro, Department of Psychology, University of Wisconsin, Madison, Wisconsin 53706.

The following two experiments study the illusion as it depends on four stimulus variables: the absolute size of the center circle, which should affect s; the size of the surrounding context circles, which should affect s^*; the distance between the center and context circles, which should affect w; and the number k of context circles.

Method

The S was instructed to judge the size of a single center circle surrounded by zero, two, four, or six other circles. Responses were made by rotating a wheel that presented single comparison circles (without surrounding circles), ranging in diameter from 8.5 to 21.5 mm. in steps of .5 mm. These 27 comparison circles were visible, 1 at a time, through a window in the wheel. The wheel was flat on the table as were the stimulus cards. Between trials, E turned the wheel haphazardly to a new starting position. The stimulus cards were 20.5 cm. square, of white tagboard. For Exp. I, the stimulus and comparison circles were solid black; for Exp. II, they were outline figures.

Experiment I.—The size of the center circle, and the number and size of surrounding context circles, were covaried in a 2 × 3 × 5 factorial design. The center circle could have diameters of 13 or 17 mm. There were two, four, or six context circles, symmetrically located in each case. The diameters of the context circles differed from the center circle by 8, 4, 0, −4, or −8 mm. for each size of center circle.

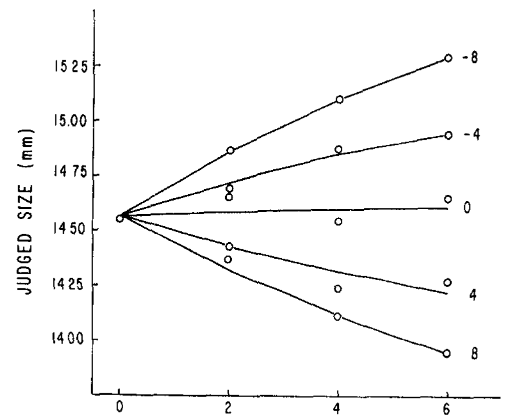

FIG. 2. Mean diameter of comparison circle judged equal to center circle of Ebbinghaus figure as a function of number of surrounding context circles. (Size difference, context circle minus center circle, listed by each curve. Data averaged over both sizes of center circle. Theoretical curves from Equation 3.)

The distance between proximal edges of center and context circles was always 6 mm. Each center circle was also presented alone, so there was a total of 32 stimulus figures presented in four separately randomized blocks each day, for 2 days, for each S. The 10 Ss were members of the university community who were paid for their services.

Experiment II.—Each figure consisted of a center circle surrounded by six context circles, all in outline form. The diameter of the context circles was 5 or 9 mm. The diameter of the center circle was 13, 15, or 17 mm. The distance between the proximal edges of center and context circles was 3, 6, 12, or 24 mm. Thus, the complete design was a 2 × 3 × 4 factorial with 24 stimuli. These were presented six times in successive randomized blocks to each of 24 paid Ss run individually in a single session.

Results

Experiment I.—Figure 2 shows illusion magnitude as a function of two stimulus variables. The response variable is mean diameter of the comparison circle judged equal to the center circle. The two top curves show that the center circle is judged larger when surrounded by small circles; the two bottom curves show that the center circle is judged smaller when surrounded by large circles. The vertical spacing of the curves indicates that the illusion is a roughly linear function of the size difference between center and context circles; the negative slope reflects contrast. The trend along the curves indicates that the illusion is a growth function of the number of surrounding context circles.

These results agree qualitatively with the judgment model of Equation 3. For a more precise description, Equation 3 was fit to the data using an iterative computer search with a least-squares criterion. Both w and s were free parameters, the latter to allow for any constant error. In addition, s^* was evaluated separately for each size of context circle.

These parameter estimates are in Table 1. Since the context circles were always presented in diametric pairs, w was evaluated for a single pair of context circles. For that case, the judgment is determined 94.3% by the center circle, 5.7% by the context. Equation 3 then implies that two and three pairs of context circles control 10.8% and 15.4% of the judgment, respectively. In particular, the effect of the

TABLE 1

PARAMETER ESTIMATES FROM EQUATION 3, EXPERIMENT I

Parameter	Estimate
w	.943
s	14.6
$s^*(+8)$	12.2
$s^*(+4)$	13.2
$s^*(\ 0)$	14.8
$s^*(-4)$	16.0
$s^*(-8)$	17.3

context circles is not simply additive, but rather follows a law of diminishing returns as shown by Fig. 2.

Two other aspects of the data need mention. Figure 2 is averaged over the two sizes of center circle, both of which showed the same pattern of data. However, the illusory effect was about 20% greater for the larger center circle, significant as shown by the Size of Center Circle × Size of Context Circles interaction, F (4, 36) = 3.61. This is consistent with previous findings that illusion magnitude is positively related to size of test figure (Waite & Massaro, 1970). In addition, Fig. 2 shows a constant error since the mean judgment with zero context circles is 14.6 mm., whereas the mean diameter of the two center circles was 15 mm. The cause of this constant error is unknown, but it may reflect comparative judgment effects induced by the window in the response wheel, and other background stimuli.

Experiment II.—The mean responses are shown in Fig. 3 as a function of the three stimulus variables. The downward trend of the curves shows that the size of the illusion decreases as the surrounding context circles move further from the center circle. The plotted curves are theoretical, obtained by fitting Equation 2 with s^* and $1 - w$ corresponding to the joint effect of the six context circles. The values of w were .800, .841, .914, and .977 for the four successive distances. The value of .841, for the 6-mm. distance, agrees closely with the value of .846 for six context circles at the same distance in Exp. I.

Figure 3 also shows that the effect of distance between center and context is largest for the largest center circle. This is shown by the greater vertical drop in the top panel, and agrees with the judgmental model. According to Equation 2, the illusory effect is $(1 - w)(s^* - s)$, and $(s^* - s)$ is a positive increasing function of size of center circle. This effect appears in the statistical analysis as a significant Context Distance × Center Size interaction, F (6, 138) = 9.54.

The last empirical result concerns the effect of context size, reflected in the differences between the paired curves of Fig. 3. The effect of context size is largest for the smallest, 13-mm. center circle, and near zero for the largest, 17-mm. center circle. Statistically, this appeared as a significant Size of Center Circle × Size of Context Circles interaction, F (2, 46) = 9.64.

This interaction was surprising, being contrary both to theoretical expectation and to extrapolation from the data of Exp. I. It may reflect a ceiling effect, further reduction in context size becoming ineffec-

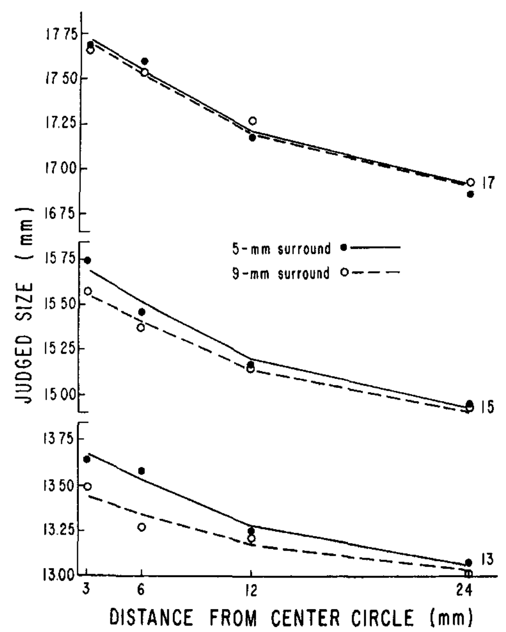

FIG. 3. Mean diameter of comparison circle judged equal to center circle of Ebbinghaus figure as a function of distance from center circle to surrounding context circles. (Size of center circle listed at the right of each pair of curves. Theoretical curves from Equation 2.)

tive after a certain point. It may also be a judgmental effect, stemming from the restriction in Exp. II to context circles smaller than the center circle. For the largest center circle, *S*s may have considered both context circles to be "small," while still making a more exact comparison for the smallest center circle. Both possibilities are speculative, of course, but can easily be tested.

Finally, Table 2 shows the estimated values for the absolute and relative sizes of the center circles for Exp. II. The estimates of absolute size, s, in the first row are nearly equal to the actual physical size. The estimates of relative size, s^* for the two sizes of context circles are in the two lower rows. These estimates reflect the trends in Fig. 3. The relative size of the smallest center circle is larger for the smaller context circle, 16.6 compared to 15.3 mm. This difference is less for the medium center circle and near zero for the largest center circle.

Discussion

The present model for comparative judgment has provided a reasonably good description of the Ebbinghaus illusion as a function of several stimulus determinants. These include the number, size, and distance of the context circles, as well as the size of the center circle itself. The only qualification required by the present data is the apparent ceiling effect for small context circles and large center circle in Exp. II. However, this effect may still be amenable to a judgmental explanation as noted. Conceptually, therefore, the present results agree with previous work on a judgmental approach to illusions (e.g., Anderson, 1970a; Massaro & Anderson, 1970; Waite & Massaro, 1970).

TABLE 2

Estimates of Absolute Size and Relative Size of Center Circle as a Function of Sizes of Center and Context Circles, Experiment II

Parameter	Size of center circle (mm.)		
	13	15	17
s	13.0	14.8	16.8
$s^*(5)$	16.6	19.2	21.4
$s^*(9)$	15.3	18.4	21.3

Note.—Symbols: s = absolute size, s^* = relative size.

Various previous investigations of the Ebbinghaus figure have also employed some kind of judgmental approach. Wapner and Werner (1957), arguing from a developmental principle of increasing differentiation of the stimulus field, predicted an increase in the Ebbinghaus illusion and a decrease in the Müller-Lyer illusion as a function of age. Both predictions were verified.

Cooper and Weintraub (1970) also tested a cognitive interpretation of the Ebbinghaus figure. The context circles were presented first, followed by the center circle after empty intervals of 0–7.5 sec., and the magnitude of the illusion decreased with larger interstimulus intervals. This decrease could be interpreted as a decrease in the salience of the comparison stimulus which, in the present model, would be represented directly as a decrease in the weight parameter for the context stimuli.

Cooper and Weintraub (1970) also present other data that may raise some problems for the judgmental view. They found similar effects from the Ebbinghaus figure and from a figure in which only the 90° arc of the context circles nearest the center circle was used. A comparison process between center circle and context arc seems less plausible than between center circle and context circle. In addition, allowing eye movements appeared to decrease the illusion, contrary to what might be expected from judgmental comparisons, though this condition was confounded with other variables. It also should be noted that their companion data on concentric circles seem more amenable to interpretation in terms of sensory than judgmental processes.

Two other developments have certain similarities to the present formulation, both conceptually and in quantitative form. The work of Hake and his associates (e.g., Hake, Faust, McIntyre, & Murray, 1967; Hake, Rodwan, & Weintraub, 1966; Rodwan, 1968; Weintraub, 1971) is similar in its emphasis on a "mixed model" in which perception depends on a composite of absolute and relative stimulus factors. A further parallel is the use of a linear integration model. The multiple discriminant methodology, on which their formulation is based, has the considerable advantage of being directly applicable to categorical response data. Like standard multiple regression, however, it ordinarily requires a prior metric on the stimulus variables.

An important difference between the multiple discriminant methodology and the present approach can be illustrated in the experiment of Hake et al. (1967). The *S*s judged the size of an inner square surrounded by a variable outer square, and a contrast effect was obtained. The discriminant function assigns equal weight to the various outer squares, regardless of their size. In contrast, the present formulation would allow for unequal weighting of different size outer squares, analogous to the differential weighting observed in the present Exp. II of the surrounding circles as a function of their distance from the center circle. Similar considerations would apply to the contrast effect observed by Rodwan (1968) in judgments of mouth size in schematic faces.

Of more immediate relevance is the work of Restle (Merryman & Restle, 1970; Restle & Merryman, 1968), based on Helson's (1964) theory of adaptation level, which emphasizes the dependence of judgment on context. Since the judgment of any stimulus is relative to the adaptation level defined by the prevailing stimulation, this formulation would be expected to make many of the same qualitative predictions as the present model. The apparent ceiling effect in Exp. II might present more difficulty for adaptation-level theory since it relies on the physical stimulus metric. In contrast, integration theory is based on the subjective values of functional measurement.

A clearer comparison between integration theory and adaptation-level theory could be obtained from judgmental tasks that produce greater contrast. In judgments of numerosity, for example, Helson and Kozaki (1968) varied the number of dots in an anchor stimulus presented before each of several test stimuli and obtained sizable contrast effects. The curves for the various anchors, plotted as a function of the number of dots in the test stimulus, formed a fan of diverging straight lines. Adaptation-level theory apparently does not account for the general shape of these curves. The present comparative judgment model (Anderson, 1970a, Equation 5) predicts that the several anchor curves should have the general bilinear form, in agreement with the data.

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